Roll No. ....

OLE-24002

B. Tech. 1st Semester (Common for All Branches) Examination – April, 2021

MATHEMATICS-I

Paper : Math-101-F

Time : Three Hours] ***[Maximum Marks :100***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in total by selecting ***one*** question from each Section. Question No. **1** is ***compulsory***.

1. (a) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$.

(b) If A and B are orthogonal matrices, prove that AB is also orthogonal.

(c) Expand $\log \sin x$ in powers of $(x-3)$.

(d) Define Beta and Gamma functions.

SECTION – A

2. Discuss the convergence of the series :

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\ldots\ldots..$$

3. Test the convergence and absolute convergence of the series :

$$\frac{1}{2(\log 2)^p}-\frac{1}{3(\log 3)^p}+\frac{1}{4(\log 4)^p}-\ldots\ldots..\infty (p>0).$$

SECTION – B

4. (a) Find the rank of the matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) For what values of parameters λ and μ do the system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution (ii) unique solution (iii) more than one solution ?

5. Find the Eigen values, Eigen vectors and verify Cayley Hamilton theorem for the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

SECTION – C

6. (a) If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

(b) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $7\frac{13}{16}$.

7. (a) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

(b) Evaluate :

$$\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx \ (a \geq 0)$$

by applying differentiation under the integral sign.

SECTION – D

8. (a) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy\, dx$.

(b) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \; dy \, dx$ and hence evaluate the same.

9. Evaluate $\iiint_R (x^2 + y^2 + z^2) \, dx \, dy \, dz$, where R denotes the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = a, (a > 0)$.